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ANALYSIS OF AXIAL STRAIN INDUCED BY 3-D CONSOLIDATION OF COHESIVE SOILS

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SUMMARY

This paper presents a simplified approach for the analysis of axial strain induced by three-dimensional consolidation of cohesive soils. The axial strain is divided into a constant volume component and a consolidation component. A relevant undrained stress–strain relationship is required to determine the constant volume component. A theoretical formulation is developed for the evaluation of the consolidation component. Predictions of the axial strain accompanying true triaxial laboratory tests for a variety of stress patterns correlate sufficiently well with the measured data. The proposed method is potentially applicable in conjunction with a finite difference scheme to analyze the time-dependent response of pile groups subjected to static vertical loading. © 1998 John Wiley & Sons, Ltd.

Key words: consolidation; cohesive soil; clay; axial strain; true triaxial; finite difference method

INTRODUCTION

Consolidation of clay was one of the principal topics of Terzaghi's pioneering work on establishing modern soil mechanics. Since then, it has received extensive attention from many geotechnical engineers. Most recently, Duncan[1] addressed this issue at the 27th Terzaghi Lecture, in which the limitations of conventional analysis were explicitly illustrated by two extraordinary field cases. Further study is clearly required to enhance our understanding and prediction of the consolidation problem. With respect to three-dimensional consolidation problem, the theory developed by Biot[2] is mostly referred to. Nevertheless, some deficiencies have also been reported.[3,4] Previous work on the application of the Biot theory was well reviewed by Smith and Hobbs,[4] mostly in conjunction with a finite element scheme. In this paper, an alternative approach is presented for the evaluation of the axial strain in any of the normal stress direction of clays induced by three-dimensional consolidation with potential application in a finite difference scheme. Two sets of laboratory data are used to verify the proposed method.

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Pile groups are commonly used deep foundations in geotechnical engineering. A substantial amount of effort has been made to evaluate the ultimate capacity and settlement of a pile group. However, time-dependent response of a pile group due to consolidation of soils surrounding the pile group has rarely been analysed. The general form of the solution is rather complex and involves the solution of (1) a stress equilibrium problem of pile–soil–pile interactions, and (2) a three-dimensional pore pressure propagation problem, with interaction through time. For the propagation problem, the finite difference method (FDM) provides an efficient and accurate estimate of the volumetric strain due to consolidation; however, it does not yield an answer for the vertical strain accompanying the volumetric strain. The method outlined below provides a reasonable approach of calculating this vertical strain.

CONCEPT AND FORMULATION

FDM generally provides an excellent estimate of the total volumetric strain; however, it does not yield an answer to the vertical strain accompanying the volumetric strain. Many ways are available of evaluating the vertical strain accompanying consolidation. On one end of the spectrum, a complicated constitutive model can be developed to quantify it directly. On the other end of the spectrum, a simplified empirical relationship can be used to estimate the vertical strain. The concept described below is somewhere between the two extremes, but probably closer to the second case. The method proposed includes two parts of axial strain. One component is calculated using the 'vertical strain factor', which is derived based on a non-linear elastic soil model. The other component is calculated based on the non-linear undrained stress–strain relationship. The derivation of the vertical strain factor and the proposed method of evaluating axial strain induced by three dimensional consolidation are discussed below.

Vertical strain factor

For time increment Δt, the volumetric strain due to consolidation $\Delta\varepsilon_v(\Delta t)$ is some proportion of the ultimate volumetric strain $(\Delta\varepsilon_v)_{ult}$. If the vertical strain increment $\Delta\varepsilon_z(\Delta t)$ induced by consolidation is assumed to be the same proportion of the ultimate vertical strain increment $(\Delta\varepsilon_z)_{ult}$, then the vertical strain increment can be evaluated as below.

$$\Delta\varepsilon_z(\Delta t) = (\Delta\varepsilon_z/\Delta\varepsilon_v)_{ult} \times \Delta\varepsilon_v(\Delta t) = \mathrm{VS} \times \Delta\varepsilon_v(\Delta t) \tag{1}$$

where VS is defined as the vertical strain factor. To evaluate the vertical strain factor, two assumptions are employed. They are (1) total stress state does not change appreciably with drainage, and (2) volume change due to shear stresses is neglected.

The ultimate vertical strain increment is the difference between the strain at the end of consolidation (ε_{zf}) and the strain at the beginning of the consolidation (ε_{zi}) , i.e.

$$(\Delta\varepsilon_z)_{ult} = \varepsilon_{zf} - \varepsilon_{zi} \tag{2}$$

Substitution of drained soil parameters for ε_{zf} and undrained soil parameters for ε_{zi}, equation (2) becomes

$$(\Delta\varepsilon_z)_{ult} = \Delta\sigma_z\left(\frac{1}{E_d} - \frac{1}{E_u}\right) - (\Delta\sigma_x + \Delta\sigma_y)\left(\frac{\nu_d}{E_d} - \frac{\nu_u}{E_u}\right) \tag{3}$$

where E_d and E_u are drained and undrained soil modulus, ν_d and ν_u are drained and undrained Poisson's ratios. $\Delta\sigma_x$, $\Delta\sigma_y$, and $\Delta\sigma_z$ represent total stress changes in the x, y, and z directions. Similar equations can be obtained for ultimate axial strain in the x and y directions. Thus, the ultimate volumetric strain $(\Delta\varepsilon_v)_{ult}$ can be obtained by summing up the ultimate axial strain in all three directions, i.e.

$$(\Delta\varepsilon_v)_{ult} = (\Delta\sigma_x + \Delta\sigma_y + \Delta\sigma_z)\left(\frac{1}{E_d} - \frac{1}{E_u}\right) - 2(\Delta\sigma_x + \Delta\sigma_y + \Delta\sigma_z)\left(\frac{\nu_d}{E_d} - \frac{\nu_u}{E_u}\right) \tag{4}$$

Dividing equation (3) by equation (4) leads to the following vertical strain factor equation:

$$\mathrm{VS} = (\Delta\varepsilon_z/\Delta\varepsilon_v)_{ult} = \frac{(\Delta\sigma_z(1 - R_e) - (\Delta\sigma_x + \Delta\sigma_y)(\nu_d - R_e\nu_u))}{(\Delta\sigma_x + \Delta\sigma_y + \Delta\sigma_z)((1 - R_e) - 2(\nu_d - R_e\nu_u))} \tag{5}$$

where $R_e = E_d/E_u$. Axial strain factors in the x and y directions can be derived in the similar way.

For triaxial test conditions, where $\Delta\sigma_z = \Delta\sigma_1$ and $\Delta\sigma_x = \Delta\sigma_y = \Delta\sigma_3$, equation (5) can be simplified as below:

$$\mathrm{VS} = \frac{((1 - R_e) - 2K_c(\nu_d - R_e\nu_u))}{(1 + 2K_c)((1 - R_e) - 2(\nu_d - R_e\nu_u))} \tag{6}$$

where $K_c = \Delta\sigma_3/\Delta\sigma_1$.

If the soil is a linear isotropic elastic material, one can set $E_u = 3G$, $E_d = 2G(1 + \nu)$, $\nu_u = 0{\cdot}5$, and $\nu_d = \nu$, then the VS calculated using equation (5) or (6) will always equal one-third, as expected. However, real soil behaviour showed that the factor VS shall vary with loading pattern. Therefore, a non-linear elastic soil model is adopted. For a non-linear elastic model, the shear modulus may vary with deformation. In this case, the ratio of the moduli, R_e, is not constrained by the expression above for the linear material. In other words, E_u and E_d are then no longer constrained by the condition that G is constant.

Two hypothetical example sets of vertical strain factors calculated for laboratory triaxial loading of different stress increment ratio (K_c) by using equation (6) are given in Figure 1. For these two demonstration examples, soil parameters E_d, E_u, ν_d, and ν_u are determined according to

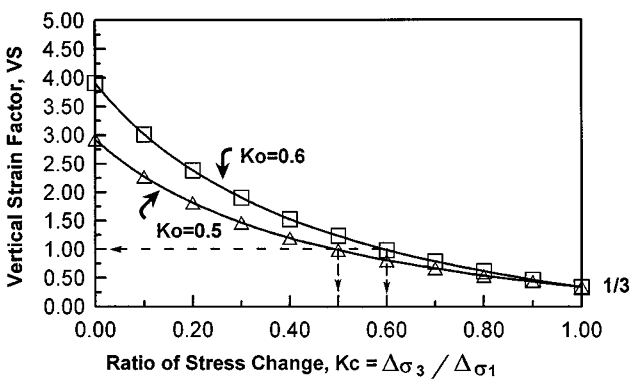


Figure 1. Vertical strain factors for anisotropic consolidation

the K_0-condition. Under K_0-condition zero soil strain in the horizontal direction is imposed such that $\nu_d = K_0/(1 + K_0)$. Lambe and Whitman[5] defined a constrained modulus D as σ_z/ε_z and derived its formulation, i.e. $D = (E_d(1 - \nu_d))/((1 + \nu_d)(1 - 2\nu_d))$. The constrained modulus can be evaluated in terms of laboratory measured soil properties such as the initial void ratio, the compression index, and the average confining stress. Thus, the value of E_d corresponding to a specific K_0 value can be estimated in terms of D and ν_d. Undrained Poisson's ratio ν_u is approximately 0·5. Undrained modulus E_u is determined based on the relationship that E_u is proportional to the confining pressure. One set of vertical strain factors is calculated using soil parameters consistent with $K_0 = 0{\cdot}5$. Soil parameters used for the $K_0 = 0{\cdot}5$ set are $\nu_d = 0{\cdot}333$, $\nu_u = 0{\cdot}5$, and $R_e = 0{\cdot}0281$. The other set is calculated using soil parameters that are representative of the $K_0 = 0{\cdot}6$ condition. The soil parameters for this set are $\nu_d = 0{\cdot}375$, $\nu_u = 0{\cdot}5$, and $R_e = 0{\cdot}0232$.

Notice that calculated vertical strain factors shown in Figure 1 decreases with increasing stress change ratio, K_c. For both conditions, the vertical strain factor converges to one-third when $K_c = 1{\cdot}0$, i.e. isotropic consolidation. The vertical strain factor is about 1·0 for $K_c = K_0$, i.e. one-dimensional consolidation. Overall, the calculated results are in good agreement with the observed laboratory behaviour.[6] On the basis of the above discussion, the vertical strain factor VS appears to be a unique function of the stress change ratio K_c. This conclusion is also consistent with the findings reported by Roscoe and Poorooshasb[7] after a study on a series of anisotropic triaxial consolidation tests.

The vertical strain factor formulation described above appears to give reasonable estimates. For certain stress paths (for example, $K_c = -0{\cdot}5$), however, the formulation given in equation (6) may have a singularity. This incremental method is also highly sensitive if the amount of stress change is insignificant. Therefore, a more general algorithm is developed in the following section. Stress tensors are also used to generalize its application in any of the normal stress directions.

Axial strain evaluation

A soil element sitting at a point A in $(S, I_\sigma/3)$ space is considered as shown in Figure 2. The task involves calculating the axial strain accompanying the subsequent drained loading, e.g. the loading path from point A to point C. The parameter S is related to the second invariant of the deviatoric stress by the following equation:

$$S = \sqrt{3J_{2d}} \tag{7}$$

where J_{2d} is the second invariant of the deviatoric stress tensor. However, for triaxial test conditions, i.e. $\sigma_2 = \sigma_3$, S is simply the deviator stress, $\sigma_1 - \sigma_3$. The parameter I_σ in the horizontal axis is the first invariant of the stress tensor. For triaxial test conditions, the parameter along the horizontal axis is simply $(\sigma'_1 + 2\sigma'_3)\ /3$.

Direct application of equation (5) for the evaluation of the axial strain accompanying the drained loading path, **AC**, yields

$$\Delta\varepsilon_a = \mathrm{VS}(\mathbf{AC}; \Delta\sigma' s) \times \Delta\varepsilon_v \tag{8}$$

where $\Delta\varepsilon_a$ and $\Delta\varepsilon_v$ represent the axial strain and the volumetric strain, respectively. VS is the strain factor given in equation (5) in terms of stress changes. The axial strain in any of the normal stress directions can be evaluated by substituting relevant stress changes into equation (5).

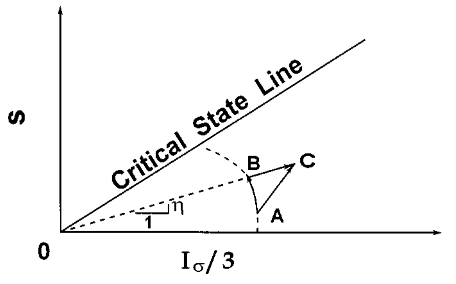


Figure 2. Sketch of a typical drained loading path

In the proposed method, the axial strain $\Delta\varepsilon_a$ for drained loading **AC** is decomposed into a constant volume component, $(\Delta\varepsilon_a)_{\mathbf{AB}}$, and a consolidation component, $(\Delta\varepsilon_a)_{\mathbf{BC}}$, such that

$$\Delta\varepsilon_a = (\Delta\varepsilon_a)_{\mathbf{AB}} + (\Delta\varepsilon_a)_{\mathbf{BC}} \tag{9}$$

where the constant volume component $(\Delta\varepsilon_a)_{\mathbf{AB}}$ can be determined using the deviatoric stress–axial strain data obtained from laboratory triaxial consolidated–undrained (CU) tests. Since the stress–strain curve can be non-linear, the proposed method for the evaluation of the axial strain accompanying drained loading is also non-linear. With the aid of some scaling laws, this method can be used to evaluate the axial strain for various loading and unloading patterns.

The consolidation component $(\Delta\varepsilon_a)_{\mathbf{BC}}$ can be evaluated by the following equation:

$$(\Delta\varepsilon_a)_{\mathbf{BC}} = \mathrm{VS}(\mathbf{BC}; \Delta\sigma' s) \times \Delta\varepsilon_v \tag{10}$$

Notably the loading path **BC** has a constant slope η. For triaxial tests with $\sigma'_2 = \sigma'_3$, the slope η can be expressed as follows:

$$\eta = \frac{\Delta\sigma'_1 - \Delta\sigma'_3}{(\Delta\sigma'_1 + 2\Delta\sigma'_3)/3} \tag{11}$$

Equation (11) can be rewritten as

$$\eta = \frac{3(1 - K_c)}{1 + 2K_c} \tag{12}$$

where $K_c = \Delta\sigma'_3/\Delta\sigma'_1$

Recalling equation (6) and previous discussion, the vertical strain factor VS is unique for a stress path of constant slope η (i.e. constant K_c). Because the slope η of path BC equals to that of path OC, the vertical strain factor for path BC is the same as that for path OC, i.e.

$$\mathrm{VS}(\mathbf{BC}; \Delta\sigma' s) = \mathrm{VS}(\mathbf{OC}; \sigma' s) \tag{13}$$

Substitution of the above relationship into equation (10) leads to

$$(\Delta\varepsilon_a)_{\mathbf{BC}} = \mathrm{VS}(\mathbf{OC}; \sigma' s) \times \Delta\varepsilon_v \tag{14}$$

The vertical strain factor VS(**OC**;$\sigma's$) can be determined using equation (5) in terms of the expected final stress state $\sigma's$.

$$\mathrm{VS}(\mathbf{OC};\sigma's) = \frac{(\sigma_z(1 - R_e) - (\sigma_x + \sigma_y)(v_d - R_e v_u))}{(\sigma_x + \sigma_y + \sigma_z)((1 - R_e) - 2(v_d - R_e v_u))} \tag{15}$$

The use of the final stress state in calculating the vertical strain could avoid the singularity and the sensitivity problem associated with small stress changes if incremental stress change is used. Therefore, the procedure illustrated by equations (9), (14) and (15) can be used to calculate the vertical strain (the z direction) induced by three-dimensional consolidation. Axial strains in other directions can be easily obtained by substituting proper stress components into equation (15).

COMPARISON WITH LABORATORY RESULTS

Data set I

Experimental data obtained from consolidated drained triaxial tests conducted on two natural soils designated as Clay X and Clay Y were used. The experimental data were published in the Proceedings of the Workshop on Limit Equilibrium. Plasticity and Generalized Stress–Strain in Geotechnical Engineering.[8] For both soils, the triaxial tests were conducted on prismatic samples trimmed from block samples. All samples were 100 per cent saturated. Experimental data selected from those tests having a negligible amount of volumetric strain were used to determine the constant volume component of the axial strain, as similar to $(\Delta\varepsilon_a)_{\mathbf{AB}}$ in equation (9). Those results are plotted in Figure 3 for Clay X and Clay Y.

One of the laboratory test results conducted on Clay X, along with predictions using the vertical strain factor method are illustrated in Figure 4. Volumetric strains measured during the test, along with the deviatoric stress–axial strain relationship given in Figure 3(a) are used to make the predictions. Soil properties required for prediction are derived from given test data. For this case, the stress path has an incremental stress ratio $K_c = \Delta\sigma_3/\Delta\sigma_1 \equiv -0{\cdot}5$, such that a singularity exists in equation (6). Therefore, making predictions using the incremental vertical strain factor equation is impossible. However, the proposed method outlined by equation (9), (14) and (15) accurately predicts axial strains in the σ_1 direction as shown in Figure 4(b).

Predictions are given in Figure 5 for a different loading pattern conducted on Clay Y. Similarly, volumetric strains measured and the deviatoric stress–axial strain relationship given in Figure 3(b) were used to make the predictions. Apparently, the proposed method gives much better results than the incremental formulation (equation (5)). The prediction could be further improved provided that a better deviatoric stress and strain relationship could be obtained.

Data set II

Results obtained from true triaxial tests of Taipei silty clay, a dark-grey low plasticity clay (CL), were used to verify the proposed method to a greater extent. Undisturbed tube samples were taken from about 10 m below the ground surface. Next, soil samples were trimmed for laboratory testing. Index properties are: LL $= 35{\cdot}2-42{\cdot}5\%$, PL $= 18{\cdot}2-28{\cdot}3\%$, $G_s = 2{\cdot}70-2{\cdot}74$, $\omega = 38{\cdot}6$ –

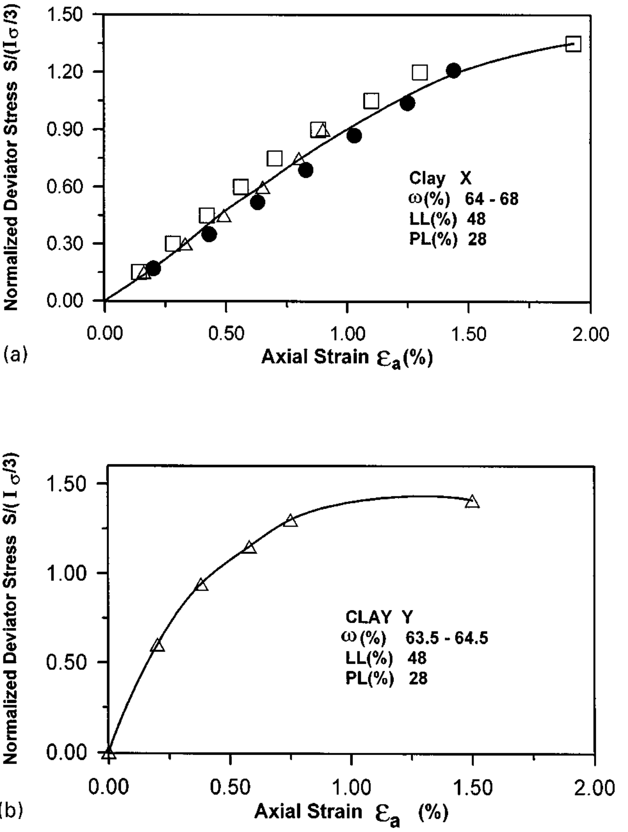


Figure 3. Normalized undrained deviator stress vs. axial strain for (a) clay *X* (b) clay Y

45·0%. Preconsolidation pressure estimated from one-dimensional consolidation tests is about 108·0 – 117·7 kPa. Both undrained and drained true triaxial tests of various stress paths were conducted at the National Taiwan Institute of Technology using the true triaxial apparatus developed by the University of Colorado.[9] All soil samples were saturated and isotropically consolidated to 147·2 kPa before true triaxial testing.

Calculated axial strains are compared with the measured values of the drained tests. Undrained test data of the stress paths studied are plotted together in Figure 6 to determine the constant volume component of the axial strain in equation (9), $(\Delta\varepsilon_a)_{AB}$. Volumetric strains measured during the drained test are used to calculate the consolidation component in equation (9), $(\Delta\varepsilon_a)_{BC}$. Predicted axial strains along the major principal stress direction for three typical stress paths compare rather reasonably with the measured data, as shown in Figures 7–9. Additionally, the extensive axial strain along the minor principal stress direction can also be predicted reasonably, as given in Figure 10. For this case, the constant volume component $(\Delta\varepsilon_a)_{AB}$ is extensive and overshadows the compressive consolidation component $(\Delta\varepsilon_a)_{BC}$, thereby resulting in extensive axial strains.

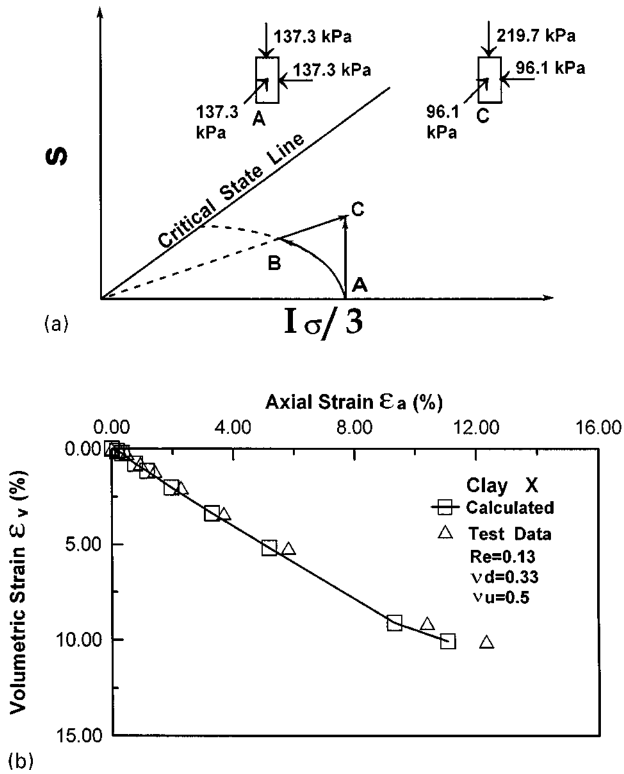


Figure 4. (a) Studied drained loading path for clay X; (b) Measured and computed axial strain accompanying the studied drained loading for clay X

DISCUSSION

The precision of the predicted axial strain relies heavily on the generalized deviatoric stress–strain data, upon which the constant volume component of the calculated axial strain is based. The best approach entails performing undrained tests of the same stress path on the same soils, as has been performed in the second verification case. Formulations such as the hyperbolic model can also be employed to facilitate the calculation.

One important parameter for the calculation of the consolidation component of the axial strain is the modulus ratio R_e, which is the ratio of the drained soil modulus and the undrained one. Undrained soil modulus used in the preceding analysis are determined from the laboratory CU tests. Secant modulus corresponding to 50% ultimate strain (ε_{50}) are used. Likewise, the secant modulus corresponding to ε_{50} of the laboratory CD tests are determined as the drained soil modulus. Although some variation in soil modulus is usually encountered, the variation of the modulus ratio is significantly reduced. Furthermore, the constant volume component constitutes a significant portion of the axial strain in many cases. Therefore, the influence of the deviation of the modulus ratio on the calculated axial strain is further limited. If the proposed method is

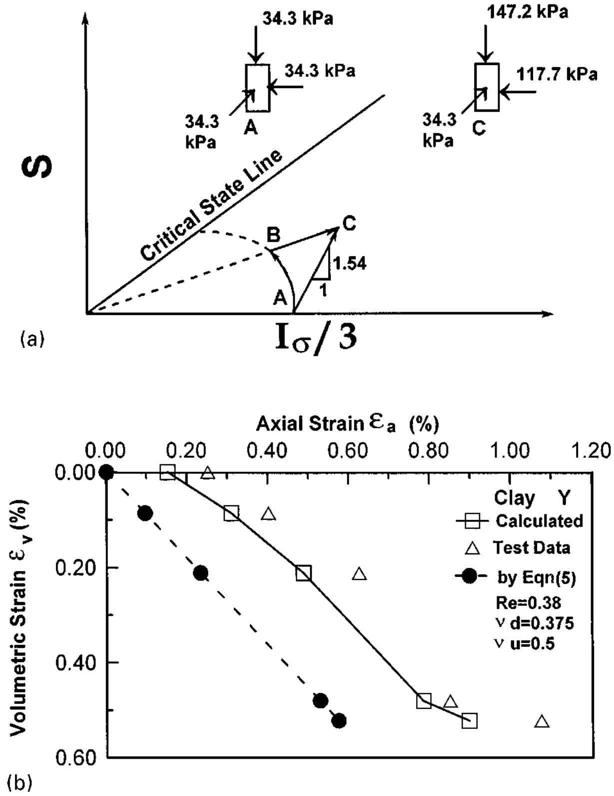


Figure 5. (a) Studied drained loading path for clay Y; (b) Measured and computed axial strain accompanying the studied drained loading for clay Y

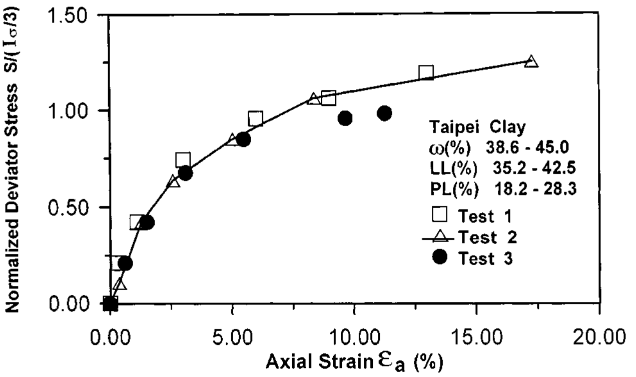


Figure 6. Normalized undrained deviator stress vs. axial strain for Taipei clay

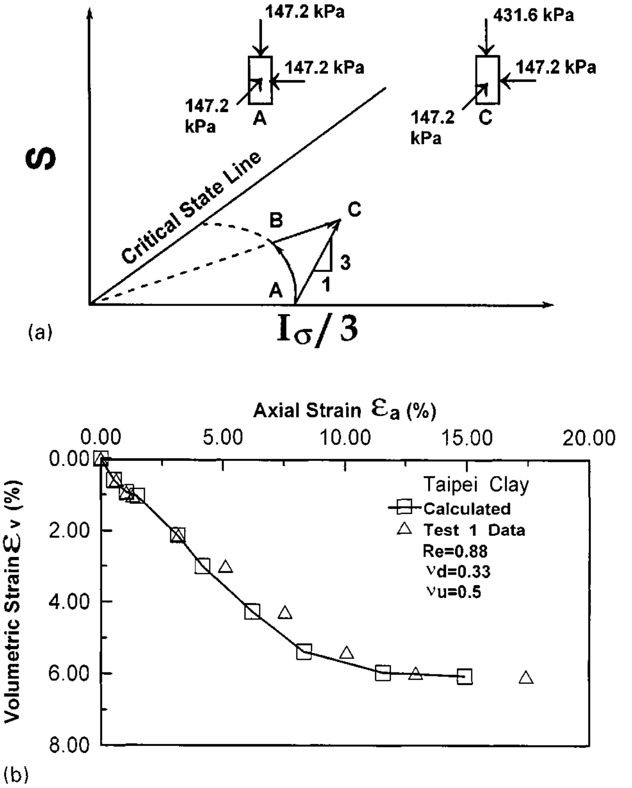


Figure 7. (a) Studied drained loading path I for Taipei clay; (b) Measured and computed axial strain accompanying the drained loading path I for Taipei clay

applied to a field case such as a time-dependent pile group analysis, the soil modulus should be determined over the range of stress changes which actually occur in the field.

Undrained Poisson's ratio of saturated clay is unique and close to 0·5. Drained Poisson's ratio can have some variation (about 0·3 to 0·4 for saturated clay). However, the deviation is limited, thereby resulting in limited impact on the outcome. For the above two data sets verified, Poisson's ratios and soil modulus are assumed to be constant throughout the calculation for each stress path for simplicity. However, strain-dependent soil properties can be easily incorporated into the numerical algorithm. If so, the accuracy of the predicted axial strain can be further enhanced. Soil samples studied are normally consolidated to slightly overconsolidated clay with little anisotropy. Application of the proposed scheme for highly overconsolidated and anisotropic soils require further analysis and experimental verification.

The proposed method can be incorporated into a finite difference scheme to study the time-dependent response of pile groups subjected to vertical loading. Preliminary study provides some qualitative insights. The analytical results indicate that the proposed method in conjunction with a finite difference scheme has the potential of providing a reasonable account of the time-dependent behaviour of the pile cap settlement, pile load distribution, and shear stress

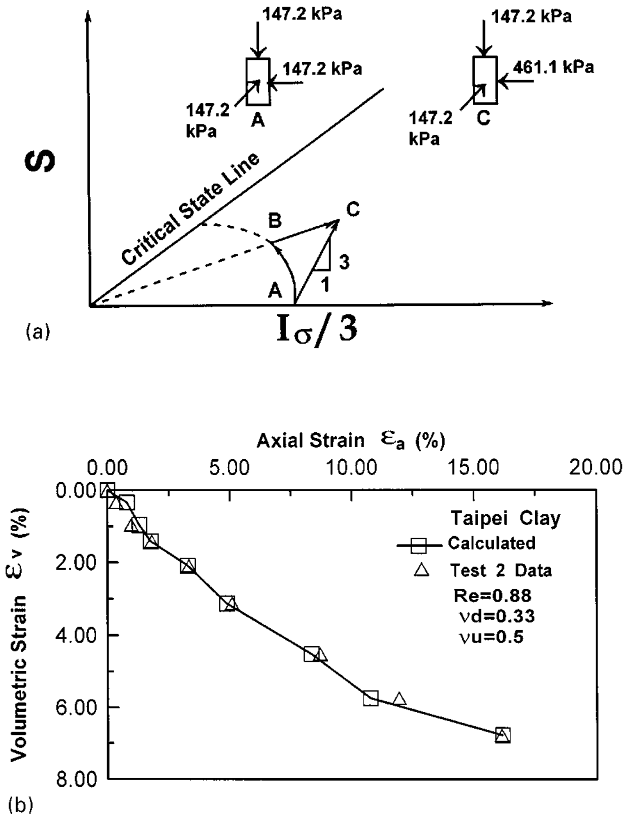
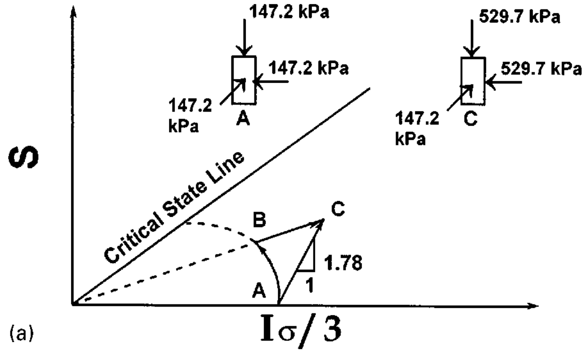


Figure 8. (a) Studied drained loading path II for Taipei clay; (b) Measured and computed axial strain accompanying the drained loading path II for Taipei clay

distribution along the pile shaft.[10] However, due to the lack of field data for verification, quantitative discussion is not included in this paper.

CONCLUSIONS

The proposed method outlined in this paper has been demonstrated to be capable of yielding reasonable predictions of the axial strain along any of the normal stress direction of laboratory soil samples subjected to true triaxial loading of various stress patterns. The axial strain can be calculated by superposition of a constant volume component and a consolidation component. The accuracy of the prediction mainly relies on (1) the accuracy of the relevant undrained deviatoric stress–strain data required to determine the constant volume component, and (2) the representativeness of the soil modulus and Poisson's ratio required to calculate the consolidation component. Inevitably, quality laboratory testing and a good sense of soil mechanics are required to ensure representative input soil parameter and thus accurate predictions. The proposed method seems to have good potential of incorporating into a finite difference scheme for the



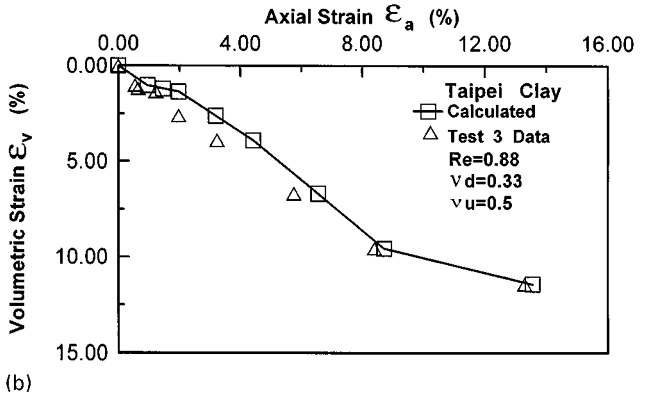


Figure 9. (a) Studied drained loading path III for Taipei clay; (b) Measured and computed axial strain accompanying the drained loading path III for Taipei clay

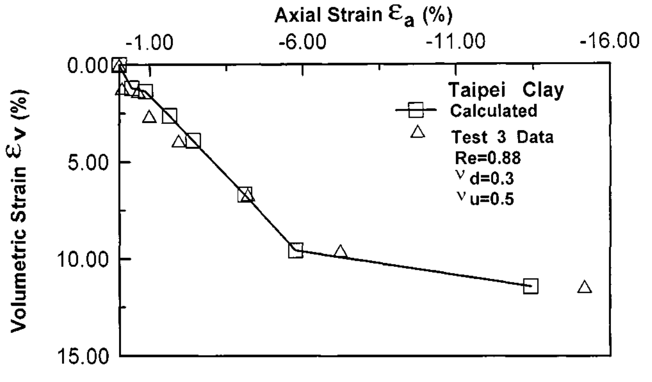


Figure 10. Measured and computed tensile axial strain accompanying the drained loading path III for Taipei clay

evaluation of the time-dependent response of pile groups. However, quantitative verification is not yet possible due to lack of field test data.

NOTATION

D	constrained modulus of soil
E_d	drained Young's modulus of soil
E_u	undrained Young's modulus of soil
G	shear modulus of soil
G_s	specific gravity of soil
I_σ	first invariant of stress tensor
J_{2d}	second invariant of stress tensor
K_c	ratio of stress change ($\Delta\sigma_3/\Delta\sigma_1$)
K_0	the coefficient of lateral earth pressure at rest
LL	liquid limit of soil
PL	plastic limit of soil
R_e	modulus ratio (E_d/E_u)
S	generalized deviator stress ($\sqrt{3J_{2d}}$)
VS	vertical strain factor

Greek letters

$\Delta\varepsilon_a$	axial strain increment
$\Delta\varepsilon_v$	volumetric strain increment
$\Delta\varepsilon_z$	vertical strain increment
$(\Delta\varepsilon_v)_{ult}$	ultimate volumetric strain increment
$(\Delta\varepsilon_z)_{ult}$	ultimate vertical strain increment
η	stress path slope
v_d	drained Poisson's ratio of soil
v_u	undrained Poisson's ratio of soil
σ_i	total principal stress ($i = 1, 2, 3$)
σ'_i	effective principal stress ($i = 1, 2, 3$)
$\Delta\sigma_i$	total principal stress change ($i = 1, 2, 3$)
$\Delta\sigma'_i$	effective principal stress change ($i = 1, 2, 3$)
$\Delta\sigma_x$	stress change in x direction
$\Delta\sigma_y$	stress change in y direction
$\Delta\sigma_z$	stress change in z direction
ω	water content of the soil

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